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Relating the Cosmological Constant and Supersymmetry Breaking in Warped Compactifications of IIB String Theory

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ABSTRACT

It has been suggested that the observed value of the cosmological constant is related to the supersymmetry breaking scale M_{susy} through the formula $\Lambda \sim M_p^4 (M_{susy}/M_p)^8$. We point out that this relation naturally arises in the codimension two solutions of warped space-time varying compactifications of string theory in which non-isotropic stringy moduli induce a small but positive cosmological constant.

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Recently, we addressed the question of whether de Sitter space [1] can be obtained from string theory [2]. Such non-singular, non-static spacetimes fall into the class of codimension two non-supersymmetric string vacua studied in Refs. [3, 4, 5]⁵. In these models, supersymmetry is explicitly broken by a global cosmic brane [6] with a core of size ℓ , extended along the $D - 2$ “longitudinal” directions. While Refs. [3, 4, 5] considered a flat Minkowski brane, the main point of Ref. [2] is the existence of non-supersymmetric codimension two solutions with a positive cosmological constant, Λ_b , in the $D - 2$ dimensional longitudinal space.

Since the cosmological constant in our model [2] is directly related to the non-isotropy of matter, we may compare with various attempts to incorporate Mach’s principle in string theory [7, 8] as well with the idea that supersymmetry breaking might have a cosmological origin [9, 10]. In particular, it has been suggested that the observed value of the cosmological constant, $\Lambda_4 \sim 10^{-44} \text{ GeV}^4$, [11, 12] may be related to the supersymmetry breaking scale M_{susy} through the formula [9]:

$$\Lambda_4 \sim M_p^4 \left(\frac{M_{susy}}{M_p} \right)^8, \quad (1)$$

with $M_{susy} \sim 10 \text{ TeV}$ and $M_p \sim 10^{19} \text{ GeV}$ as appropriate in 4-dimensions⁶. It is the aim of this note to point out that the set-up of Ref. [2] leads naturally to Eq. (1). In particular, the stringy moduli induce a non-trivial relation between the scale of the global cosmic brane, ℓ , the non-isotropy of matter, ω , induced by the brane, and Λ_b [2]. We take this observation one step further and find an explicit relation between Λ_b , ℓ and the natural mass scales in this theory, M_D and M_{D-2} , the Planck scales in the bulk and along the brane, respectively, thus deriving an equation analogous to Eq. (1).

Although the detailed physics leading to this relation is unclear, we find it very intriguing that Eq. (1) emerges naturally in our framework. Still, one of the most important unresolved questions in the scenario presented in Ref. [2] is the issue of the stability of this non-supersymmetric background in full string theory. Since supersymmetry is broken, one also has to address the effects of stringy corrections. We will argue that those corrections are negligible.

The general framework of our analysis is as in Refs. [3, 4, 5, 2], to which we refer the reader for a more detailed analysis. We consider Type IIB string theory (compactified on a

⁵By string vacua we denote solutions that satisfy the corresponding type IIB supergravity equations of motion and contain moduli with proper $SL(2, \mathbb{Z})$ properties.

⁶The essential ingredient of this proposal is a conjectured relevance of a non-decoupling between the microscopic and macroscopic degrees of freedom [8, 9, 12] for the cosmological constant problem. This conjecture is natural from the following intuitive perspective on the cosmological constant problem. On one hand, the cosmological constant is tied to the fundamental physics of the vacuum, because Λ_4 is essentially given by the vacuum energy density. On the other hand, the cosmological constant is related to the large scale behavior of the universe, since a small cosmological constant implies that the observable universe is big and almost flat.

fixed supersymmetry preserving space) in which the axion-dilaton system, (α, ϕ) , described by the complex modulus field $\tau = \alpha + i \exp(-\phi)$, varies over the $x_{D-2, D-1}$ directions of the uncompactified spacetime. Thus, the relevant part of the low-energy effective D -dimensional action of the modulus, τ , coupled to gravity reads

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} (R - \mathcal{G}_{\tau\bar{\tau}} g^{\mu\nu} \partial_\mu \tau \partial_\nu \bar{\tau} + \dots) . \quad (2)$$

Here $\mu, \nu = 0, \dots, D-1$, $2\kappa^2 = 16\pi G_N^{(D)}$, where $G_N^{(D)}$ is the D -dimensional Newton constant, and $\mathcal{G}_{\tau\bar{\tau}} = -(\tau - \bar{\tau})^{-2}$ is the metric on the complex structure moduli space of a torus⁷.

Let us now briefly review the codimension two solution with positive cosmological constant, Λ_b , along the longitudinal direction of the cosmic brane [2]. The metric Ansatz is:

$$ds^2 = A^2(z) \bar{g}_{ab} dx^a dx^b + \ell^2 B^2(z) (dz^2 + d\theta^2) , \quad (3)$$

$$\bar{g}_{ab} dx^a dx^b = -dx_0^2 + e^{2\sqrt{\Lambda_b} x_0} (dx_1^2 + \dots + dx_{D-3}^2) , \quad (4)$$

where $z = \log(r/\ell)$. As in the case when $\Lambda_b = 0$, we find that the explicit solutions for τ are aperiodic, such as $\tau = \alpha_0 + i g_s^{-1} \exp(i\omega\theta)$, but do exhibit a non-trivial $SL(2, \mathbb{Z})$ monodromy [3]⁸. This ensures our solution to be stringy (although classical and non-supersymmetric) rather than merely a supergravity vacuum. Note in particular that the dilaton of the Type IIB superstring theory varies with the polar angle, not the radial distance. Also, modular invariance forces $g_s \sim O(1)$ in this case. We will return to discussing the corrections to our classical solution.

Following [2], the Einstein equation can be simplified

$$R_{\mu\nu} = \mathcal{G}_{\tau\bar{\tau}} \partial_\mu \tau \partial_\nu \bar{\tau} \stackrel{\text{def}}{=} \tilde{T}_{\mu\nu} . \quad (5)$$

Since the metric (3) is axially symmetric, while τ is independent of the radial distance from the cosmic brane, $\tilde{T}_{\mu\nu} = \text{diag}[0, \dots, 0, \frac{1}{4}\omega^2 \ell^{-2}]$. Eq. (5) then defines the general class of our spacetimes as *almost* Ricci-flat: $R_{\mu\nu} = \text{diag}[0, \dots, 0, \frac{1}{4}\omega^2 \ell^{-2}]$, where $\omega^2 > 0$ is indeed related to supersymmetry breaking [3] and ℓ is the (transversal) length scale of the global cosmic brane.

The $R_{ab} = 0$ part of Eq. (5) reduces to a single equation, giving:

$$B^2 = \ell^{-2} \Lambda_b^{-1} \left(A'^2 + \frac{1}{(D-3)} A A'' \right) = \ell^{-2} \Lambda_b^{-1} \frac{h'' h^{-\frac{D-4}{D-2}}}{(D-2)} , \quad (6)$$

⁷Recall that because of its $SL(2, \mathbb{Z})$ properties, the axion-dilaton, τ , can be thought of as the complex structure of a T^2 , in analogy with F-theory [13].

⁸Although $\partial\tau$ does not transform correctly under $SL(2, \mathbb{Z})$ transformations, it is straightforward to show that $|\partial\tau|^2$, which appears in the action (2), is invariant.

which determines $B(z)$ in terms of $A(z)$ or $h(z) \stackrel{\text{def}}{=} A(z)^{D-2}$. With this substitution, the remaining components of Eq. (5) reduce to the following single equation:

$$\frac{1}{2(D-2)} \frac{h'^2}{h^2} - \frac{h''}{2h} + \frac{h'h'''}{2hh''} = -\frac{1}{8}\omega^2. \quad (7)$$

For $\omega \neq 0$ ($\tau \neq \text{const.}$), Eq. (7) has a perturbative, analytic solution⁹:

$$\begin{aligned} A(z) &= Z(z) \left(1 - \frac{\omega^2 \rho_0^2 (D-3)}{24(D-1)(D-2)} Z(z)^2 + O(\omega^4) \right), \\ B(z) &= \frac{1}{\ell \rho_0 \sqrt{\Lambda_b}} \left(1 - \frac{\omega^2 \rho_0^2}{8(D-1)} Z(z)^2 + O(\omega^4) \right), \end{aligned} \quad (8)$$

where $Z(z) = 1 - z/\rho_0$ and $\rho_0 > 0$. As was shown in Refs. [2], close to the horizon spacetime is asymptotically flat in agreement with the behavior of Rindler space [15].

In contrast, when $\Lambda_b = 0$ the solution is very different [3],

$$\tilde{A}(z) = \tilde{Z}(z)^{\frac{1}{(D-2)}}, \quad \tilde{B}(z) = \tilde{Z}(z)^{\frac{-(D-3)}{2(D-2)}} e^{\frac{\xi}{2a_0}(1-\tilde{Z}(z)^2)}, \quad (9)$$

where now $\tilde{Z} = (1 - a_0 z)$, and we restrict to $a_0 > 0$. This solution exhibits a naked singularity, at $z = a_0^{-1}$ ($\tilde{Z} = 0$), for the global cosmic brane.

While the naked singularity has been removed by $\Lambda_b > 0$, it was shown [2] that the global cosmic brane solution (9) is still a good approximation to Eq. (8) away from the horizon. In particular, by comparing Eq. (8) with Eq. (9) close to the core it was shown that

$$a_0 \approx 0.9 (D-2) \rho_0^{-1}, \quad \xi \approx \frac{1}{0.9} \omega^2 \rho_0^2 \frac{\rho_0^{-1}}{8(D-2)}, \quad (10)$$

$$\Lambda_b \approx \frac{\omega^2}{2(D-2)\ell^2}, \quad \text{and} \quad \omega \approx 0.9 \sqrt{2(D-2)} \rho_0^{-1}. \quad (11)$$

Thus, the cosmological constant is directly related to the non-trivial variation of the matter as a function of θ ! This gives a very non-trivial relation between the stringy moduli, and hence string theory itself, and a positive Λ_b .

Finally, the Newton constant, $G_N^{(D-2)} = M_{D-2}^{-(D-4)}$, in $D-2$ dimensions and the zero-mode wave function normalization, $\langle \psi_0 | \psi_0 \rangle$, are [2]:

$$G_N^{(D-2)} = M_D^{-(D-2)} \langle \psi_0 | \psi_0 \rangle^{-1}, \quad \text{and} \quad \langle \psi_0 | \psi_0 \rangle \sim \frac{\pi}{D-3} \frac{\ell}{\sqrt{\Lambda_b}}. \quad (12)$$

The volume of the transversal space, $\langle \psi_0 | \psi_0 \rangle$, is large [2] and drives the large M_{D-2}/M_D hierarchy. This then implies the following relation,

$$\Lambda_{D-2} \sim \left(\frac{\pi}{D-3} \right)^2 M_{D-2}^{D-2} (\ell M_{D-2})^2 \left(\frac{M_D}{M_{D-2}} \right)^{2D-4}, \quad (13)$$

⁹This solution is of the same form as that discussed by Gregory [14] for the $U(1)$ vortex solution.

where $\Lambda_{D-2} = \Lambda_b / G_N^{(D-2)}$ is the energy density in $D-2$ dimensions.

Recall that ℓ is the characteristic (transverse) size of the cosmic brane, for the formation of which no concrete physical mechanism is known. However, should ℓ be stabilized by a longitudinal $D-2$ -dimensional physics mechanism, then $\ell \sim M_{D-2}^{-1}$ and

$$\Lambda_{D-2} \sim \left(\frac{\pi}{D-3} \right)^2 M_{D-2}^{D-2} \left(\frac{M_D}{M_{D-2}} \right)^{2D-4}. \quad (14)$$

The original scenario of Ref. [2] then applies, where the 10-dimensional spacetime of the Type IIB string theory is compactified on a 4-dimensional supersymmetry preserving space¹⁰ of characteristic size $M_{10}^{-1} = M_6^{-1} \sim (10 \text{ TeV})^{-1} \sim 10^{-19} \text{ m}$. The cosmic brane of Ref. [2] then describes a 3+1-dimensional de Sitter world-brane, with the characteristic scale $M_4 \sim 10^{19} \text{ GeV}$. Eq. (11) in this case implies that $\omega \sim \ell \sqrt{\Lambda_b} \sim 10^{-21}$. Furthermore, since $\Lambda_b \sim 10^{-82} \text{ GeV}^2$, Eq. (11) implies that $L \stackrel{\text{def}}{=} \ell \omega^{-1} \sim 10^{40} \text{ GeV}^{-1} \sim 10^{24} \text{ m}$, which coincides with the Hubble radius.

Alternatively, if ℓ is stabilized by the “bulk” D -dimensional physics, then $\ell \sim (M_D)^{-1}$ and

$$\Lambda_{D-2} \sim \left(\frac{\pi}{D-3} \right)^2 M_{D-2}^{D-2} \left(\frac{M_D}{M_{D-2}} \right)^{2D-6}. \quad (15)$$

In this case, the original scenario of Ref. [2] needs a minor modification: the 10-dimensional spacetime of the Type IIB string theory is first compactified on a 3-dimensional supersymmetry preserving space of characteristic size $M_{10}^{-1} = M_7^{-1} \sim (10 \text{ TeV})^{-1} \sim 10^{-19} \text{ m}$. The cosmic brane of Ref. [2] then describes a 4+1-dimensional de Sitter world-brane, with the characteristic scale $M_5 \sim 10^{19} \text{ GeV}$. Upon a Kaluza-Klein compactification on a circle of radius $M_5^{-1} \sim (10^{19} \text{ GeV})^{-1} \sim 10^{-35} \text{ m}$, this yields a 3+1-dimensional de Sitter world-brane with $M_4 = M_5 \sim 10^{19} \text{ GeV}$. Eq. (11) now implies that $\omega \sim \ell \sqrt{\Lambda_b} \sim 10^{-36}$.

Note that Eqs. (14) with $D=6$, and (15) with $D=7$ respectively immediately imply the desired relationship (1):

$$\Lambda_4 \sim M_p^4 \left(\frac{M_{\text{susy}}}{M_p} \right)^8, \quad (16)$$

upon identifying $M_{10} = M_D$ with the scale of supersymmetry breaking, and $M_{D-2} = M_4$ with the four dimensional Planck scale, M_p . As far as we know, this is the first time such a relation between the observed value of the cosmological constant and the scale of supersymmetry breaking has been obtained in a specific dynamical situation. Note that this relation crucially depends on the fact that the zero mode normalization scales as $\langle \psi_0 | \psi_0 \rangle \sim \frac{\ell}{\sqrt{\Lambda_b}}$, which is a specific feature of the scenario presented in [2].

¹⁰All remaining supersymmetry will be broken by the cosmic brane solution [3].

In fact, there exists a whole spectrum of scenarios, albeit with powers of the mass scale ratio in Eqs.(14)–(15) other than 8 in Eq. (1). These scenarios differ in the compactification/cosmic brane Ansatz sequencing. For example, for a codimension two cosmic brane in 10 dimensions with ℓ stabilized by the longitudinal 8-dimensional physics, Eq. (14) applies: $\Lambda_8 \sim M_8^8 (\frac{M_{10}}{M_8})^{16}$. After wrapping on a suitable 4-dimensional space (of size M_8^{-1}), for the desirable values of $\Lambda_4 \sim 10^{-44} \text{ GeV}^4$ and $M_4 = M_8 \sim 10^{19} \text{ GeV}$, we find that $M_{10} \sim 5.6 \times 10^6 \text{ GeV}$ is the fundamental scale. At the opposite end, by compactifying the 10-dimensional spacetime on a suitable 4-dimensional space and then constructing a codimension two cosmic brane in 6 dimensions with ℓ stabilized by the “bulk” 6-dimensional physics, Eq. (15) applies: $\Lambda_4 \sim M_4^4 (\frac{M_6}{M_4})^6$. For the desirable values of $\Lambda_4 \sim 10^{-44} \text{ GeV}^4$ and $M_8 = M_4 \sim 10^{19} \text{ GeV}$, the fundamental scale becomes $M_6 = M_{10} \sim 100 \text{ MeV}$.

Finally, let us conclude by discussing the stringy and quantum corrections to our solution. We will assume that the D -dimensional theory has the equivalent of $N = 4$ supersymmetry in 4-dimensions, or equivalently 16 supercharges. This will always be the case as long as we are considering type II theories with at most a K3-compactification from ten to six dimensions. First, note that $\Lambda_b \sim \omega^2/\ell^2$ is consistent with the notion that supersymmetry breaking and a non-zero cosmological constant are related. In particular, from Ref. [4], the supersymmetry breaking is indicated by the non-vanishing of $\frac{dA}{dr}$. Owing to Eqs. (10) and (11), this is well estimated by $A' \sim \omega \ell^{-1} \sim \sqrt{\Lambda_b}$, which is a very small parameter. This, we argue, justifies neglecting the corrections due to supersymmetry breaking¹¹. The α' corrections due to the global cosmic string would have to take the form $\alpha'/\langle\psi_0|\psi_0\rangle$ where the denominator is the volume of the transverse space and $\alpha' \sim M_D^{-2}$ is the string scale. From Eq. (12) it then follows that the string corrections are very small. Now, although our solution is not BPS, supersymmetry is broken very weakly. Therefore, the corrections should be proportional both to the coupling and the supersymmetry breaking parameter. Since $g_s \sim O(1)$ because of modular invariance [3], $g_s \not\rightarrow \infty$, and the smallness of the supersymmetry breaking parameter justifies neglecting strong coupling corrections.

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¹¹In general there will be α' and string coupling corrections without breaking supersymmetry; these will not be considered here.

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